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Relacions entre les raons trigonomètriques

Per a qualsevol angle agut α es compleixen aquestes relacions:

$$\operatorname{tg} \alpha = \frac{\sin \alpha}{\cos \alpha} \quad \sin^2 \alpha + \cos^2 \alpha = 1 \quad \operatorname{tg}^2 \alpha + 1 = \frac{1}{\cos^2 \alpha}$$

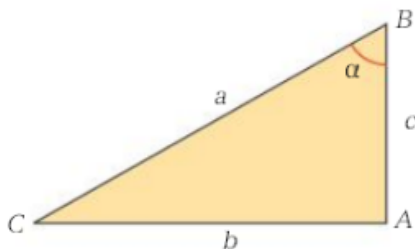
DEMOSTRACIÓ DEL LLIBRE:

$$\bullet \operatorname{tg} \alpha = \frac{\sin \alpha}{\cos \alpha} \rightarrow \frac{\sin \alpha}{\cos \alpha} = \frac{\frac{b}{a}}{\frac{c}{a}} = \frac{b \cdot \cancel{a}}{c \cdot \cancel{a}} = \frac{b}{c} = \operatorname{tg} \alpha$$

$$\bullet \sin^2 \alpha + \cos^2 \alpha = \left(\frac{b}{a}\right)^2 + \left(\frac{c}{a}\right)^2 = \frac{b^2}{a^2} + \frac{c^2}{a^2} = \frac{b^2 + c^2}{a^2} \stackrel{a^2 = b^2 + c^2}{=} \frac{a^2}{a^2} = 1$$

- Si dividim la igualtat $\sin^2 \alpha + \cos^2 \alpha = 1$ entre $\cos^2 \alpha$, obtenim:

$$\frac{\sin^2 \alpha}{\cos^2 \alpha} + \frac{\cos^2 \alpha}{\cos^2 \alpha} = \frac{1}{\cos^2 \alpha} \rightarrow \operatorname{tg}^2 \alpha + 1 = \frac{1}{\cos^2 \alpha}$$



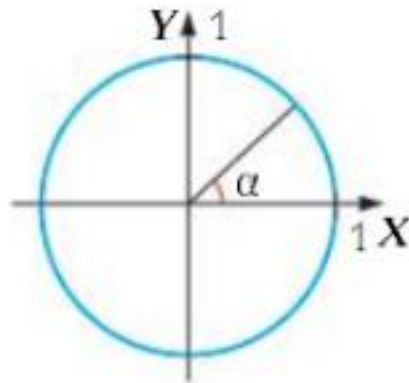
HO ESCRIVIM AIXÍ

$$(\sin \alpha)^2 = \sin^2 \alpha$$

$$(\cos \alpha)^2 = \cos^2 \alpha$$

$$(\operatorname{tg} \alpha)^2 = \operatorname{tg}^2 \alpha$$

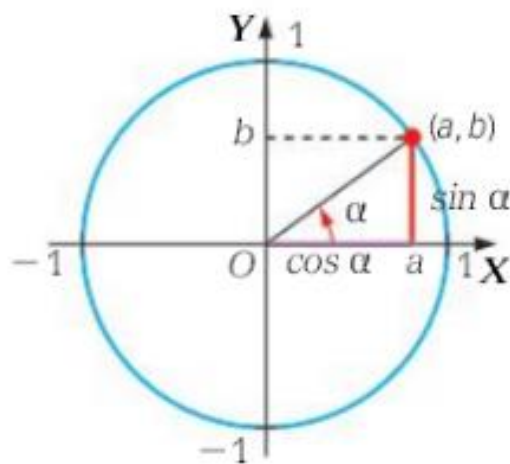
DEMOSTRACIÓ AMB CIRCUMFERÈNCIA GONIOMÈTRICA: (radi = 1)



Circumferència goniomètrica

La **circumferència goniomètrica**, o **circumferència unitat**, és una circumferència de radi 1 centrada a l'origen de coordenades.

Per representar un angle α situem el vèrtex a l'origen de coordenades i un dels costats sobre l'eix X . En mesurem l'amplitud en sentit contrari a les busques del rellotge (sentit positiu).



El sinus i el cosinus d'un angle qualsevol adopten valors entre -1 i 1 . La tangent pot adoptar qualsevol valor.

Aplicant el Teorema de Pitàgores al triangle de la circumferència goniomètrica:

$$\sin \alpha = \frac{b}{1} = b \quad \cos \alpha = \frac{a}{1} = a$$

$$\sin^2 \alpha + \cos^2 \alpha = 1$$



Calcular totes les raons trigonomètriques d'un angle agut coneixent una de les raons

Troba totes les raons trigonomètriques de l'angle α sabent que:

a) $\sin \alpha = 0,98$

b) $\operatorname{tg} \alpha = 0,84$

Passos que cal seguir

1. Fem servir les fórmules trigonomètriques que relacionen la raó que ens dona l'enunciat amb la resta de raons.

2. Resolem el sistema que en resulta, en què les incògnites són les raons trigonomètriques que ens falten.

$$\left. \begin{array}{l} \text{a) } \sin^2 \alpha + \cos^2 \alpha = 1 \rightarrow 0,98^2 + \cos^2 \alpha = 1 \\ \operatorname{tg} \alpha = \frac{\sin \alpha}{\cos \alpha} \rightarrow \operatorname{tg} \alpha = \frac{0,98}{\cos \alpha} \end{array} \right\}$$

$$\left. \begin{array}{l} \text{b) } \operatorname{tg}^2 \alpha + 1 = \frac{1}{\cos^2 \alpha} \rightarrow 0,84^2 + 1 = \frac{1}{\cos^2 \alpha} \\ \operatorname{tg} \alpha = \frac{\sin \alpha}{\cos \alpha} \rightarrow 0,84 = \frac{\sin \alpha}{\cos \alpha} \end{array} \right\}$$

$$\left. \begin{array}{l} \text{a) } 0,98^2 + \cos^2 \alpha = 1 \\ \operatorname{tg} \alpha = \frac{0,98}{\cos \alpha} \end{array} \right\} \rightarrow \cos \alpha = \sqrt{1 - 0,98^2} = 0,2$$

$$\operatorname{tg} \alpha = \frac{0,98}{\cos \alpha} \xrightarrow{\cos \alpha = 0,2} \operatorname{tg} \alpha = \frac{0,98}{0,2} = 4,9$$

$$\left. \begin{array}{l} \text{b) } 0,84^2 + 1 = \frac{1}{\cos^2 \alpha} \rightarrow \cos \alpha = \sqrt{\frac{1}{0,84^2 + 1}} = 0,77 \\ 0,84 = \frac{\sin \alpha}{\cos \alpha} \end{array} \right\}$$

$$0,84 = \frac{\sin \alpha}{\cos \alpha} \xrightarrow{\cos \alpha = 0,77} 0,84 = \frac{\sin \alpha}{0,77} \rightarrow \sin \alpha = 0,84 \cdot 0,77 = 0,65$$

Si α és un angle agut, quan aïllem ens quedem l'arrel positiva.

$$\sin \alpha = +\sqrt{1 - \cos^2 \alpha}$$

$$\cos \alpha = +\sqrt{1 - \sin^2 \alpha}$$

Deures - Exercicis:

Ex 10,11,12,13,14 (pàg. 139)

10 Considera que l'angle α és agut, i calcula'n el sinus a partir de les raons següents.

a) $\cos \alpha = 0,4321$

c) $\cos \alpha = 0,9531$

b) $\cos \alpha = 0,1357$

d) $\cos \alpha = 0,2864$

11 Donat un angle agut α , calcula'n el cosinus a partir de les raons següents.

a) $\sin \alpha = 0,1827$

b) $\sin \alpha = 0,9542$

c) $\sin \alpha = 0,4531$

d) $\sin \alpha = 0,7988$

12 A partir d'un angle agut α , calcula'n el sinus en aquests casos sabent que:

a) $\operatorname{tg} \alpha = 3$

b) $\operatorname{tg} \alpha = 1$

10. Pàgina 139

$$a) \sin^2 \alpha + \cos^2 \alpha = 1 \rightarrow \sin^2 \alpha = 0,4321^2 = 1 \rightarrow \sin^2 \alpha = 0,8133 \rightarrow \sin \alpha = 0,9018$$

$$b) \sin^2 \alpha + \cos^2 \alpha = 1 \rightarrow \sin^2 \alpha = 0,1357^2 = 1 \rightarrow \sin^2 \alpha = 0,9816 \rightarrow \sin \alpha = 0,9908$$

$$c) \sin^2 \alpha + \cos^2 \alpha = 1 \rightarrow \sin^2 \alpha = 0,9531^2 = 1 \rightarrow \sin^2 \alpha = 0,0916 \rightarrow \sin \alpha = 0,3027$$

$$d) \sin^2 \alpha + \cos^2 \alpha = 1 \rightarrow \sin^2 \alpha = 0,2864^2 = 1 \rightarrow \sin^2 \alpha = 0,918 \rightarrow \sin \alpha = 0,9581$$

11. Pàgina 139

$$a) \sin^2 \alpha + \cos^2 \alpha = 1 \rightarrow 0,1827^2 + \cos^2 \alpha = 1 \rightarrow \cos^2 \alpha = 0,9666 \rightarrow \cos \alpha = 0,9832$$

$$b) \sin^2 \alpha + \cos^2 \alpha = 1 \rightarrow 0,9542^2 + \cos^2 \alpha = 1 \rightarrow \cos^2 \alpha = 0,0895 \rightarrow \cos \alpha = 0,2992$$

$$c) \sin^2 \alpha + \cos^2 \alpha = 1 \rightarrow 0,4531^2 + \cos^2 \alpha = 1 \rightarrow \cos^2 \alpha = 0,7947 \rightarrow \cos \alpha = 0,8915$$

$$d) \sin^2 \alpha + \cos^2 \alpha = 1 \rightarrow 0,7988^2 + \cos^2 \alpha = 1 \rightarrow \cos^2 \alpha = 0,3619 \rightarrow \cos \alpha = 0,6016$$

12. Pàgina 139

$$a) \operatorname{tg}^2 \alpha = 1 \rightarrow \frac{1}{\cos^2 \alpha} = 3^2 = 1 \rightarrow \frac{1}{\cos^2 \alpha} = 10 \rightarrow \frac{1}{\cos^2 \alpha} = \cos^2 \alpha = 0,1$$

$$\sin^2 \alpha + \cos^2 \alpha = 1 \rightarrow \sin^2 \alpha = 0,1 = 1 \rightarrow \sin^2 \alpha = 0,9 \rightarrow \sin \alpha = 0,9487$$

$$b) \operatorname{tg}^2 \alpha = 1 \rightarrow \frac{1}{\cos^2 \alpha} = 1^2 = 1 \rightarrow \frac{1}{\cos^2 \alpha} = 2 \rightarrow \frac{1}{\cos^2 \alpha} = \cos^2 \alpha = 0,5$$

$$\sin^2 \alpha + \cos^2 \alpha = 1 \rightarrow \sin^2 \alpha = 0,5 = 1 \rightarrow \sin^2 \alpha = 0,5 \rightarrow \sin \alpha = 0,7071$$

13 Sabent que l'angle α és agut, calcula en aquests casos les altres raons trigonomètriques d'aquest angle.

a) $\sin \alpha = \frac{7}{25}$

e) $\cos \alpha = \frac{8}{17}$

b) $\sin \alpha = \frac{3}{5}$

f) $\cos \alpha = \frac{12}{13}$

c) $\sin \alpha = \frac{12}{37}$

g) $\cos \alpha = \frac{20}{29}$

d) $\sin \alpha = \frac{11}{61}$

h) $\cos \alpha = \frac{5}{13}$

14 Considera que l'angle α és agut i calcula'n el sinus i el cosinus en els casos següents.

a) $\operatorname{tg} \alpha = \frac{5}{3}$

b) $\operatorname{tg} \alpha = \frac{3}{4}$

13. Pàgina 139

$$a) \sin^2 \alpha + \cos^2 \alpha = 1 - \left(\frac{7}{25}\right)^2 - \cos^2 \alpha = 1 - \frac{49}{625} - \cos^2 \alpha = 1 \rightarrow \cos^2 \alpha = \frac{576}{625} \rightarrow \cos \alpha = \frac{24}{25}$$

$$\operatorname{tg} \alpha = \frac{\sin \alpha}{\cos \alpha} \rightarrow \operatorname{tg} \alpha = \frac{\frac{7}{25}}{\frac{24}{25}} \rightarrow \operatorname{tg} \alpha = \frac{7}{24}$$

$$b) \sin^2 \alpha + \cos^2 \alpha = 1 - \left(\frac{3}{5}\right)^2 - \cos^2 \alpha = 1 - \frac{9}{25} - \cos^2 \alpha = 1 - \cos^2 \alpha = \frac{16}{25} \rightarrow \cos \alpha = \frac{4}{5}$$

$$\operatorname{tg} \alpha = \frac{\sin \alpha}{\cos \alpha} \rightarrow \operatorname{tg} \alpha = \frac{\frac{3}{5}}{\frac{4}{5}} \rightarrow \operatorname{tg} \alpha = \frac{3}{4}$$

$$c) \sin^2 \alpha + \cos^2 \alpha = 1 - \left(\frac{12}{37}\right)^2 - \cos^2 \alpha = 1 - \frac{144}{1.369} - \cos^2 \alpha = 1 - \cos^2 \alpha = \frac{1.225}{1.369} \rightarrow \cos \alpha = \frac{35}{37}$$

$$\operatorname{tg} \alpha = \frac{\sin \alpha}{\cos \alpha} \rightarrow \operatorname{tg} \alpha = \frac{\frac{12}{37}}{\frac{35}{37}} \rightarrow \operatorname{tg} \alpha = \frac{12}{35}$$

$$d) \sin^2 \alpha + \cos^2 \alpha = 1 - \left(\frac{11}{61}\right)^2 - \cos^2 \alpha = 1 - \frac{121}{3.721} - \cos^2 \alpha = 1 - \cos^2 \alpha = \frac{3.600}{3.721} \rightarrow \cos \alpha = \frac{60}{61}$$

$$\operatorname{tg} \alpha = \frac{\sin \alpha}{\cos \alpha} \rightarrow \operatorname{tg} \alpha = \frac{\frac{11}{61}}{\frac{60}{61}} \rightarrow \operatorname{tg} \alpha = \frac{11}{60}$$

$$e) \sin^2 \alpha - \cos^2 \alpha = 1 - \sin^2 \alpha - \left(\frac{8}{17}\right)^2 = 1 - \sin^2 \alpha - \frac{64}{289} = 1 - \sin^2 \alpha = \frac{225}{289} \rightarrow \sin \alpha = \frac{15}{17}$$

$$\operatorname{tg} \alpha = \frac{\sin \alpha}{\cos \alpha} \rightarrow \operatorname{tg} \alpha = \frac{\frac{15}{17}}{\frac{8}{17}} \rightarrow \operatorname{tg} \alpha = \frac{15}{8}$$

$$f) \sin^2 \alpha + \cos^2 \alpha = 1 \rightarrow \sin^2 \alpha - \left(\frac{12}{13} \right)^2 = 1 \rightarrow \sin^2 \alpha + \frac{144}{169} = 1 - \sin^2 \alpha = \frac{25}{169} \rightarrow \sin \alpha = \frac{5}{13}$$

$$\operatorname{tg} \alpha = \frac{\sin \alpha}{\cos \alpha} = \frac{\frac{5}{13}}{\frac{12}{13}} \rightarrow \operatorname{tg} \alpha = \frac{5}{12}$$

$$g) \sin^2 \alpha + \cos^2 \alpha = 1 - \sin^2 \alpha - \left(\frac{20}{29} \right)^2 = 1 \rightarrow \sin^2 \alpha - \frac{400}{841} = 1 - \sin^2 \alpha = \frac{441}{841} \rightarrow \sin \alpha = \frac{21}{29}$$

$$\operatorname{tg} \alpha = \frac{\sin \alpha}{\cos \alpha} = \frac{\frac{21}{29}}{\frac{20}{29}} \rightarrow \operatorname{tg} \alpha = \frac{21}{20}$$

$$h) \sin^2 \alpha + \cos^2 \alpha = 1 \rightarrow \sin^2 \alpha - \left(\frac{5}{13} \right)^2 = 1 - \sin^2 \alpha - \frac{25}{169} = 1 - \sin^2 \alpha = \frac{144}{169} \rightarrow \sin \alpha = \frac{12}{13}$$

$$\operatorname{tg} \alpha = \frac{\sin \alpha}{\cos \alpha} = \frac{\frac{12}{13}}{\frac{5}{13}} \rightarrow \operatorname{tg} \alpha = \frac{12}{5}$$

14. Pàgina 139

$$a) \operatorname{tg}^2 \alpha - 1 = \frac{1}{\cos^2 \alpha} - \left(\frac{5}{3} \right)^2 + 1 = \frac{1}{\cos^2 \alpha} - \frac{25}{9} - 1 = \frac{1}{\cos^2 \alpha} - \frac{34}{9} = \frac{1}{\cos^2 \alpha} - \cos^2 \alpha = \frac{9}{34} \rightarrow \cos \alpha = 0,5145$$

$$\sin^2 \alpha + \cos^2 \alpha = 1 \rightarrow \sin^2 \alpha + \frac{9}{34} = 1 \rightarrow \sin^2 \alpha = \frac{25}{34} \rightarrow \sin \alpha = 0,8575$$

$$b) \operatorname{tg}^2 \alpha - 1 = \frac{1}{\cos^2 \alpha} - \left(\frac{3}{4} \right)^2 + 1 = \frac{1}{\cos^2 \alpha} - \frac{9}{16} - 1 = \frac{1}{\cos^2 \alpha} - \frac{25}{16} = \frac{1}{\cos^2 \alpha} - \cos^2 \alpha = \frac{16}{25} \rightarrow \cos \alpha = \frac{4}{5}$$

$$\sin^2 \alpha + \cos^2 \alpha = 1 \rightarrow \sin^2 \alpha + \frac{16}{25} = 1 \rightarrow \sin^2 \alpha = \frac{9}{25} \rightarrow \sin \alpha = \frac{3}{5}$$